

OKLAHOMA STATE UNIVERSITY
SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING



**ECEN 5713 Linear Systems
Fall 2003
Final Exam**



Ph.D. Students- DO ALL FIVE PROBLEMS

MS Students- Choose FOUR out of five problems: Indicate below which four are chosen

1). _____ 2). _____ 3). _____ 4). _____

Name: _____

Student ID: _____

E-Mail Address: _____

Problem 1:

Find an canonical form realization (in minimal order) from SISO continuous-time system given below:

$$5 \ddot{y}(t) + (-1) \dot{y}(t) + -2 y(t) = 2 \ddot{u}(t) + 2 \dot{u}(t) - 2 u(t).$$

Notice that gain blocks may be dependent. Show the state space representation and its corresponding simulation diagram.

Problem 2:

- a) Prove that a square matrix is nonsingular if and only if there is no zero eigenvalue.
b) Show that functions of the same matrix commute, i.e., $f(A)g(A) = g(A)f(A)$.

Problem 3:

Show that if λ is an eigenvalue of the matrix

$$= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{-1} \end{bmatrix},$$

then a corresponding eigenvector is $= [1 \quad \lambda \quad \cdots \quad \lambda^{-1}]$.

Problem 4:

Find an “equivalent” continuous-time

dynamical equation of

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ -1 & 1 \end{bmatrix} u(k),$$

$$x(k) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}.$$

Problem 5:

Show the continuous, time-invariant system $\dot{x} = Ax + Bu$ is controllable, if and only if $\rho[\lambda I - A, B] = n$ for every eigenvalue $\lambda, i = 1, \dots, n$ of A .